LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2014**

ST 2962 - MODERN PROBABILITY THEORY

Date : 08/04/2014 Dept. No. [ ] Max. : 100 Marks
Time : 09:00-12:00

SECTION A

Answer all of the following. **10X2=20**

1. Define: σ – Algebra.
2. Define: Monotone Field.
3. Define: Discrete Probability Space.
4. Define: Lebesgue - Stieltjes Measure.
5. Define Mixture of Distributions.
6. Let X be a continuous Gamma Variate, derive, $M_x(\theta)$.
7. Derive the Mean of Beta Distribution of second kind.
8. Explain, almost surely convergence.
9. When will you say, a random variable is said to be centered at some constant c and its expectation?
10. State Markov's theorem.

SECTION B

Answer any FIVE from the following

5X8=40

11. A Field is closed under finite unions. Conversely, a class closed under complementation and finite union is a field.
12. Explain: Induced Probability Space with an example.
13. Show that Binomial Distribution converges to Poisson distribution in the sense that,

$$P[X=k]=\binom{n}{k}p^k q^{n-k} \to e^{-\lambda}\lambda^k / k! = P[Y=k]$$

If n→∞, p→0 such that np= λ, where X is binomial with index n and Y is Poisson with parameter λ.

14. Let (X, Y) ~ BVN ($\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$). Derive the marginal density function of X.
15. State and prove the properties of Expectation of Simple random variables.
16. If $X_n \xrightarrow{p} X$ then, there exists a subsequence $\{X_{nk}\}$ of $\{X_n\}$ which converges a.s. to X.
17. If $\sum_1^n x_k = S_n = S < \infty and$ $b_n \uparrow \infty$, then $\frac{1}{b_n}\sum_n^1 b_k x_k \to 0$.
18. If X_k's are independent and identically distributed r.v's, $S_n \to c$(a.s.) where c is a finite number iff E|X|<∞. Then prove that c=E(X).

SECTION C

Answer the following **2x20=40**

19. i) Let ξ_i be the class of all intervals of the form (a, b), (a<b) a,b Ɛ R, but arbitrary. Then P.T. σ (ξ_i) =Ɓ. (8)

ii) If $E|X|^r<\infty$, then prove that $E|X|^{r'}<\infty$ for $0<r'\leq r$ and $E(X^k)$ exists and is finite for $k<r$, k an integer. (6)

iii) The distribution function F_X of r.v. X is non-decreasing, continuous on the right with $F_X(-\infty) = 0$ and $F_X(+\infty) = 1$. Conversely, every function F with the above properties is the d.f. of a r.v. on some probability space. (6)

(OR)

20. i) Let $X_n \xrightarrow{L} X, Y_n \xrightarrow{L} c$, then P.T.

    a. $X_n + Y_n \xrightarrow{L} X + c$

    b. $X_n Y_n \xrightarrow{L} cX$

    c. $X_n / Y_n \xrightarrow{L} X / c$. (10)

    ii) State and prove the Monotone Convergence theorem. (10)

21. i) For a series of independent r.v's, prove that,

    a. Convergence in probability and in law are equivalent.

    b. If $|X_n|\leq b$, for some b and $E(X_n) = 0$ for all n then convergence in q.m. in probability and in law are equivalent. (10)

    ii) State and prove Kolmogorov Inequalities (10)

(OR)

22. i) State and prove Lindeberg - Feller Theorem. (12)

    ii) Let $\{x_n\}$ be a sequence of i.i.d. r.v.'s with characteristic function $\varphi(u)$. Then prove that $S_n/n \xrightarrow{p} E(X)$ (8)